Decentralized control synthesis with decentralized observer using orthogonal functions

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Abstract—In this paper, a new technique of decentralized control synthesis with decentralized state observer is proposed by using orthogonal functions. The use of this interesting tool allows the conversion of differential state equations into a set of algebraic ones by expanding the system inputs and outputs variables on an orthogonal functions basis and using the operational properties of the considered orthogonal functions. The developed method leads to the determination of decentralized control gains even when the subsystems states are not all measurable. With this approach each controlled subsystem of the global system has the desired performances of a chosen reference model.

Index Terms—Decentralized control, decentralized state observer, orthogonal functions, reference model.

I. INTRODUCTION

In the last decades The decentralized control has given rise to enourmous studies based on large scale interconnected systems, especially when it is related to sensitive fields such as power generating plants, aircraft dynamics, economic models and others.

The decentralized control of an interconnected system aims to make each system perfectly regulated using only its own local state variables, and at the same time to insure the global stability of the whole system. Often the complete state measurements are not available at each subsystem for decentralized control. Consequently the state observer can be used to estimate the non measurable subsystem states [1].

However, the complexity of the considered large scale interconnected systems makes, in general, the synthesis of a decentralized controller relatively difficult, espacially when desired performances have to be imposed for the system as optimal control, or the tracking of a chosen reference model.

In the literature different techniques of decentralized control design are developed [2], [3], [4]. Only that, the proposed approaches are in general concerned with particular classes of interconnected systems, and specific conditions have to be verified in order to achieve the problem resolution.

The focus of this paper is the development of a decentralized control design technic with decentralized observer using the expension of the dynamic system and the state observer variables on an orthogonal functions basis.

The orthogonal functions have been succesfuly applied for the identification, model reduction, analysis and control of linear and some classes of nonlinear systems [5], [6].

There is different orthogonal functions basis such as Legendre polynomials [7], Chebychev [8], or Hermite polynomials [9] and Walsh functions [10].

The important operational properties of the orthogonal functions as the integration operational matrix are exploited in this contribution to design a decentralized observer and develop a new technique leading to the determination of linear decentralized control laws such that each controlled subsystem of the global system has the desired performances of a chosen reference model.

This paper is organized as follows: a short review of the orthogonal functions is persented in the second section. In the third section our main contribution is exposed where we present the proposed approach of the decentralized control synthesis with decentralized state observer using orthogonal functions. A numerical simulated example is provided in the fourth section to illustrate the developed method.

II. REVIEW OF ORTHOGONAL FUNCTIONS

Consider a complete set of orthogonal functions $\phi = \{\phi_i(\text{t}), \text{i} \in \mathbb{N}\}$ defined on an interval $[\text{a}, \text{b}] \subset \mathbb{R}$. The principle of orthogonality leads to the property:

$$\forall i,j \in \mathbb{N}, \int_a^b w(t)\phi_i(t)\phi_j(t)dt = \delta_{ij}q_i \qquad (1)$$

where $w(t)$ is the weight function δ_{ij} is the Kronecker's symbol.

An integrable function f on $[\text{a}, \text{b}]$ can be developed as:

$$f(t) = \sum_{i=0}^{\infty} f_i\phi_i(t) \qquad (2)$$

where:

$$f_i = 1/q_i \int_a^b w(t)f(t)\phi_i(t)dt \qquad (3)$$

For obvious practical reasons, the development is truncated to order N which is large enough to allow a good approximation. Thus, one has:

$$f(t) \cong \sum_{i=0}^{N-1} f_i\phi_i(t) = F_N\Phi_N(t) \qquad (4)$$

with:

$$F_N = [f_0 \quad f_1 \quad ... \quad f_{N-1}]$$

$$\Phi_N(t) = [\phi_0(t) \quad \phi_1(t) \quad ... \quad \phi_{N-1}(t)]^T$$

This truncated projection of scalar or vector functions can be very useful in practice in different kinds of engineering problems related to modelling, identification ,analysis, simulation, control, etc.

Indeed, by means of the operational properties of orthogonal functions, the differential equations describing dynamic process can be reduced into algebraic relations allowing important simplifications in the analysis or synthesis problems.

A. Operational matrix of integration

For a given basis of orthogonal functions $\phi = \{\phi_i(t), i \in \mathbb{N}\}$, the operational matrix of integration is a constant matrix $P_N \in \mathbb{R}^{N \times N}$ such as:

$$\int_a^t \Phi_N(\tau)d\tau \cong P_N \Phi_N(t) \tag{5}$$

Obviously, the operational matrix of integration depends on the type of considered orthogonal basis. We consider in this study to use a set of Legendre polynomials as a complete basis of orthogonal functions.

B. Legendre polynomials

The Legendre polynomials are orthogonal on the interval $[-1, 1]$, with a weight function $w(\tau) = 1$.

The set of Legendre polynomials is obtained from the formula of Olinde-Rodrigues:

$$L_n(\tau) = \frac{1}{2^n n!} \frac{d^n (\tau^2 - 1)^n}{d\tau^n} \tag{6}$$

This gives:

$$L_0(\tau) = 1, L_1(\tau) = \tau, L_2(\tau) = \left(\frac{3\tau^2 - 1}{2}\right) \tag{7}$$

These polynomials can also be obtained from the recursive relationship [11]:

$$(n + 1)L_{n+1}(\tau) = (2n + 1)\tau L_n(\tau) - nL_{n-1}(\tau) \tag{8}$$

with: $L_0(\tau) = 1$ et $L_1(\tau) = \tau$

C. The shifted Legendre polynomials

To obtain orthogonal Legendre polynomials on the interval $[0, t_f]$, we perform the following change of variable:

$$\tau = \frac{2t}{t_f} - 1 \quad with \quad 0 \le t \le t_f \tag{9}$$

The recursive relationship (8) becomes:

$$(n + 1)\phi_{n+1}(t) = (2n + 1)(\frac{2t}{t_f} - 1)\phi_n(t) - n\phi_{n-1}(t) \tag{10}$$

with $\phi_n(t)$ The shifted Legendre polynomials for $0 \le t \le t_f$. and $\phi_0(t) = 1$ et $\phi_1(t) = \frac{2t}{t_f} - 1$

The principle of orthogonality of The shifted Legendre polynomials is expressed by the following equation [12]:

$$\int_0^{t_f} \phi_i(t)\phi_j(t)dt = \frac{t_f}{2i + 1}\delta_{ij} \tag{11}$$

So, any integrable function on $[0, t_f]$ can be developed into a series of shifted Legendre polynomials as follows:

$$f(t) = \sum_{i=0}^{\infty} f_i\phi_i(t) \tag{12}$$

where the coefficients f_i are given by [13]:

$$f_i = \frac{2i + 1}{t_f} \int_0^{t_f} f(t)\phi_i(t)dt \tag{13}$$

we choose an order N sufficiently large to represent the function $f(t)$:

$$f(t) \cong \sum_{i=0}^{N-1} f_i\phi_i(t) = F_N \Phi_N(t) \tag{14}$$

with:

$$F_N = [f_0 \quad f_1 \quad ... \quad f_{N-1}]$$

and:

$$\Phi_N(t) = [\phi_0(t) \quad \phi_1(t) \quad ... \quad \phi_{N-1}(t)]^T$$

D. The operational matrix of integration of shifted Legendre polynomials

In the case of shifted Legendre polynomials, The operational matrix of integration P_N is given as follows [14]:

$$P_N = \frac{t_f}{2} \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\frac{1}{5} & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\frac{1}{2N-3} & 0 & \frac{1}{2N-3} \\ 0 & 0 & 0 & \cdots & 0 & -\frac{1}{2N-1} & 0 \end{pmatrix}$$

III. PROBLEM FORMULATION

Let us consider a global large scale system (S) consisting of M interconnected subsystems (S_i) described by the following state equation:

$$(S_i) \begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{\substack{j=1 \\ j \ne i}}^{M} A_{ij} x_j(t) \\ y_i(t) = C_i x_i(t) \end{cases} \tag{15}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ $u_i(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^{p_i}$ are respectively the state vector, the control vector and the output vector of the subsystem (S_i). A_i, B_i, C_i and A_{ij} are the constant matrices characterising the subsystem (S_i) with respective dimensions $(n_i \times n_i)$, $(n_i \times m_i)$, $(p_i \times n_i)$ and $(n_i \times n_j)$.

A. The decentralized state observer design

we are interested to design a decentralized linear observer that will track the state of every subsystem (S_i). Hence, the designed observer is described by the following state equations:

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iu_i(t) + L_i(y_i(t) - \hat{y}_i(t)) \\ \hat{y}_i(t) = C_i\hat{x}_i(t) \end{cases} \quad (16)$$

with:
$L_i \in \mathbb{R}^{n_i \times p_i}$: the observation gain matrix of the ith subsystem.
$\hat{x}_i$: the observed state.
The dynamic of the observation error between the ith true state and the ith observer output is given by:

$$\dot{\varepsilon}_i(t) = \dot{x}_i(t) - \dot{\hat{x}}_i(t) = (A_i - L_iC_i)\varepsilon_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij}x_j(t) \quad (17)$$

When considering the globel system (S), the observation error can be expressed by:

$$\dot{\varepsilon}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A - LC)\varepsilon(t) + Tx(t) \quad (18)$$

with:

$$A = \begin{pmatrix} A_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_M \end{pmatrix} L = \begin{pmatrix} L_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & L_M \end{pmatrix}$$

$$C = \begin{pmatrix} C_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_M \end{pmatrix} T = \begin{pmatrix} 0 & \cdots & A_{1M} \\ \vdots & \ddots & \vdots \\ A_{M1} & \cdots & 0 \end{pmatrix}$$

The integration of the equation (18) yields the following equation:

$$\int_0^t \dot{\varepsilon}(\tau)d\tau = \varepsilon(t) - \varepsilon(t = 0) =$$

$$\int_0^t (A - LC)\varepsilon(\tau)d\tau + \int_0^t Tx(\tau)d\tau \quad (19)$$

Consider now an N orthogonal functions basis $\Phi_N(t)$, then the projection of the vector $\varepsilon(t)$ gives:

$$\varepsilon(t) \cong \varepsilon(t)_N\Phi_N(t)$$

The use of these approximations in the integrated equation (19), and the exploitation of the operational matrix of integration of the orthogonal basis $\Phi_N(t)$ lead to the following algebraic equation:

$$\varepsilon_N\Phi_N(t) - \varepsilon_{0,N}\Phi_N(t) = (A - LC)\varepsilon_N P_N\Phi_N(t)$$

$$+Tx_N P_N\Phi_N(t) \quad (20)$$

Making use of the Vec operator, which transform a matrix structure into a vector and the specific property [15]:

$$Vec(ABC) = (C^T \otimes A)Vec(B) \quad (21)$$

the equation (20) yields the following one:

$$Vec(\varepsilon_N) =$$

$$(I_{n\times N} - P_N{}^T \otimes (A - LC))^{-1}(Vec(\varepsilon_{0,N}) + (P_N{}^T \otimes T)Vec(x_N)) \quad (22)$$

To determine the observation gain parameters, we choose a reference observation error described by the following equation:

$$\dot{\varepsilon}_r(t) = M_r\varepsilon_r(t) \quad (23)$$

such that the observation error should be as close as possible to the reference, this condition can be expressed by:

$$\varepsilon(t) = \varepsilon_r(t) \quad (24)$$

The projection of the equation (23) on the orthogonal functions basis, and the use of the Vec operator give the following relation:

$$\varepsilon_{r,N} = (I_{n\times N} - P_N{}^T \otimes M_r)^{-1}\varepsilon_{r0,N} \quad (25)$$

The equation (24) can be equivalent to the next one:

$$\varepsilon_N = \varepsilon_{r,N} \quad (26)$$

which may be written by:

$$\omega^{-1}(\varepsilon_{0,N} + (P_N{}^T \otimes T)Vec(x_N)) = \omega_r{}^{-1}\varepsilon_{r0,N} \quad (27)$$

where:

$$\omega = (I_{n\times N} - P_N{}^T \otimes (A - LC))$$

$$\omega_r = (I_{n\times N} - P_N{}^T \otimes M_r)$$

when considering the same initial values, the observation gain parameters can be obtained by minimizing the next system of equations δ:

$$(\delta)\begin{cases} \omega = \omega_r \\ \omega^{-1}(P_N{}^T \otimes T) = 0_{n\times N} \end{cases} \quad (28)$$

B. Decentralized control with state observer synthesis

It is desired to determine for the global system a control law with a decentralized structure of the following form:

$$u_i(t) = J_ir_i(t) - K_i\hat{x}_i(t) \quad (29)$$

where: $r_i(t) \in \mathbb{R}^{p_i}$ $i = 1, ..., M$ order vector.
$K_i \in \mathbb{R}^{m_i \times n_i}$ and $J_i \in \mathbb{R}^{m_i \times p_i}$, are the control parameters to be determined $(i = 1, ..., M)$. The subsystem (S_i) with the control law (29) can be written by the following equation:

$$\dot{x}_i(t) = A_ix_i(t) - B_iK_i\hat{x}_i(t) + B_iJ_ir(t) + \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij}x_j(t) \quad (30)$$

The integration of the equation (30) from null initial conditions yields the following equation:

$$x_i(t) = \int_0^t A_ix_i(\tau)d\tau - \int_0^t B_iK_i\hat{x}_i(\tau)d\tau$$

$$+\sum_{\substack{j=1 \\ j \neq i}}^{M} \int_0^t A_{ij}x_j(\tau)d\tau + \int_0^t B_iJ_ir_i(\tau)d\tau \quad (31)$$

The projection of vectors $x_i(t)$, $\hat{x}_i(t)$ and $r_i(t)$ on a base of orthogonal functions gives:

$$x_i(t) \cong x_{iN}\Phi_N(t)$$

$$\hat{x}_i(t) \cong \hat{x}_{iN}\Phi_N(t)$$

$$r_i(t) \cong r_{iN}\Phi_N(t)$$

The exploitation of the operational matrix of integration of the orthogonal basis $\Phi_N(t)$ in the integrated equation (31), lead to the following algebraic equation:

$$x_{iN}\Phi_N(t) = A_i x_{iN} P_N \Phi_N(t) - B_i K_i \hat{x}_{iN} + B_i J_i r_N P_N \Phi_N(t)$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij} x_{jN} P_N \Phi_N(t) \tag{32}$$

Making use of the Vec operator, the equation (32) yields the following one:

$$(I_{N \times n} - (P_N^T \otimes A_i))Vec(x_{iN}) = -(P_N^T \otimes B_i K_i)Vec(\hat{x}_{iN})$$

$$+ (P_N^T \otimes B_i J_i)Vec(r_N) + \sum_{\substack{j=1 \\ j \neq i}}^{M} (P_N^T \otimes A_{ij})Vec(x_{jN}) \tag{33}$$

when considering the equation (29), the observer state equations become:

$$\dot{\hat{x}}_i(t) = (A_i - B_i K_i - L_i C_i)\hat{x}_i(t) + B_i J_i r_i(t) + L_i C_i x_i(t) \tag{34}$$

The projection of the equation (34) on the orthogonal functions basis, and the use of the Vec operator give the following relations:

$$M(K_i, L_i)Vec(\hat{x}_{iN}) = (P_N^T \otimes B_i J_i)Vec(r_N)$$

$$+ (P_N^T \otimes L_i C_i)Vec(x_{iN}) \tag{35}$$

with:

$$M(K_i, L_i) = (I_{N \times n} - (P_N^T \otimes (A_i - B_i K_i - L_i C_i)))$$

By substitution, the equations (33) and (35) give:

$$M(A_i)Vec(x_{iN}) = M(J_i)Vec(r_N) + \sum_{\substack{j=1 \\ j \neq i}}^{M} M(A_{ij})Vec(x_{jN})$$

$$- M(K_i)M(K_i, L_i)^{-1}(M(J_i)Vec(r_N) + M(L_i)Vec(x_{iN})) \tag{36}$$

where:

$$M(A_i) = (I_{N \times n} - (P_N^T \otimes A_i))$$

$$M(J_i) = (P_N^T \otimes B_i J_i)$$

$$M(A_{ij}) = (P_N^T \otimes A_{ij})$$

$$M(K_i) = (P_N^T \otimes B_i K_i)$$

$$M(J_i) = (P_N^T \otimes B_i J_i)$$

$$M(L_i) = (P_N^T \otimes L_i C_i)$$

The equation (36) can then be expressed by the following one:

$$\alpha_{ii}Vec(x_{iN}) = \beta_i Vec(r_N) + \sum_{\substack{j=1 \\ j \neq i}}^{M} M(A_{ij})Vec(x_{jN}) \tag{37}$$

where:

$$\alpha_{ii} = M(A_i) + M(K_i)M(K_i, L_i)^{-1}M(L_i)$$

$$\alpha_{ij} = M(A_{ij})$$

$$\beta_i = M(J_i) - M(K_i)M(K_i, L_i)^{-1}M(J_i)$$

From the equation (37) one may write:

$$\underbrace{\begin{pmatrix} Vec(x_{1N}) \\ \vdots \\ Vec(x_{MN}) \end{pmatrix}}_{V_x} =$$

$$\underbrace{\begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1M} \\ \vdots & \ddots & \vdots \\ \alpha_{M1} & \cdots & \alpha_{MM} \end{pmatrix}^{-1}}_{M_A^{-1}(K_i, L_i)} \underbrace{\begin{pmatrix} \beta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_M \end{pmatrix}}_{M_B(K_i, L_i, J_i)} \underbrace{\begin{pmatrix} Vec(r_{1N}) \\ \vdots \\ Vec(r_{MN}) \end{pmatrix}}_{V_r}$$

$$\tag{38}$$

In the other hand, the projection matrices of the different subsystems outputs on the orthogonal basis $\Phi_N(t)$:

$$y_i(t) \cong y_{iN}\Phi_N(t)$$

can be expressed using the projection matrices x_{iN} of the state vector $x_i(t)$ by the following relation:

$$\underbrace{\begin{pmatrix} Vec(y_{1N}) \\ \vdots \\ Vec(y_{MN}) \end{pmatrix}}_{V_y} =$$

$$\underbrace{\begin{pmatrix} I_N \otimes C_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & I_N \otimes C_M \end{pmatrix}}_{M_C} \underbrace{\begin{pmatrix} Vec(x_{1N}) \\ \vdots \\ Vec(x_{MN}) \end{pmatrix}}_{V_x} \tag{39}$$

The Problem now is the determination of K_i and J_i matrices $(i = 1, ..., M)$. Such that each controlled subsystem (S_i) has an input-output behaviour as same as possible to the reference model described by the following state equation:

$$(R_i) \begin{cases} \dot{z}_i(t) = E_i z_i(t) + F_i r_i(t) \\ y_{r_i}(t) = G_i z_i(t) \end{cases} \tag{40}$$

where : $z_i(t) \in \mathbb{R}^{\tilde{n}_i}$ is the state vector of the ith reference submodel, and $y_{r_i}(t) \in \mathbb{R}^{p_i}$ its output vector. E_i, F_i and G_i are the chosen matrices characterising the reference model with respective dimensions $(\tilde{n}_i \times \tilde{n}_i)$, $(\tilde{n}_i \times p_i)$ and $(p_i \times \tilde{n}_i)$. The projection of the vectors $z_i(t)$ and $y_{r_i}(t)$ on the orthogonal

functions basis, and the use of the Vec operator give the following relations:

$$\begin{cases} z_i(t) \cong z_{iN}\Phi_N(t) \\ y_{r_i}(t) \cong y_{r_iN}\Phi_N(t) \end{cases} \qquad (41)$$

$$\begin{cases} Vec(z_{iN}) = H_{ii}Vec(r_{iN}) \\ Vec(y_{riN}) = \gamma_{ii}Vec(r_{iN}) \end{cases} \qquad (42)$$

with:

$$\begin{cases} H_{ii} = [I_{N \times \tilde{n}_i} - (P_N^T \otimes E_i)]^{-1}(P_N^T \otimes F_i) \\ \gamma_{ii} = (I_N \otimes G_i)H_{ii} \end{cases} \qquad (43)$$

Having the same input-output behaviour of each subsystem (S_i) $(i = 1, ..., M)$ and its reference model (R_i) can be expressed by the following relation:

$$\underbrace{\begin{pmatrix} Vec(y_{1N}) \\ \vdots \\ Vec(y_{MN}) \end{pmatrix}}_{V_y} = \underbrace{\begin{pmatrix} Vec(y_{r1N}) \\ \vdots \\ Vec(y_{rMN}) \end{pmatrix}}_{V_{yr}} \qquad (44)$$

which may be written as:

$$M_C M_A^{-1}(K_i)M_B(L_i)V_r = \Gamma V_r \qquad (45)$$

where:

$$\Gamma = \begin{pmatrix} \gamma_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_{MM} \end{pmatrix}$$

The equality (45) has to be verified for any V_r (since each subsystem (S_i) must be identical to the reference model (R_i) for any input signal $r_i(t)$), then one obtains the following equation:

$$M_C M_A^{-1}(K_i)M_B(L_i) = \Gamma \qquad (46)$$

which resolution by means of the numerical minimization of the following norm ξ:

$$\xi = \left\| M_C M_A^{-1}(K_i, L_i)M_B(K_i, L_i, J_i) - \Gamma \right\| \qquad (47)$$

yields the different control law gains K_i and J_i $(i = 1, ..., M)$. This optimization problem can be easily carried out using specific Matlab functions.

IV. ILLUSTRATIVE EXAMPLE

To illustrate the proposed technique of decentralized control design using orthogonal functions, we consider the interconnected system composed of two second order systems characterized by a state equation of the form (15) with the following matrix parameters:

$$A_1 = \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} B_1 = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} C_1 = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -2 & 0.5 \\ 1 & -1.5 \end{pmatrix} B_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} C_2 = \begin{pmatrix} 0 & 0.6 \end{pmatrix}$$

$$A_{12} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.36 \end{pmatrix} A_{21} = \begin{pmatrix} 0 & 0.2 \\ 0.54 & 0 \end{pmatrix}$$
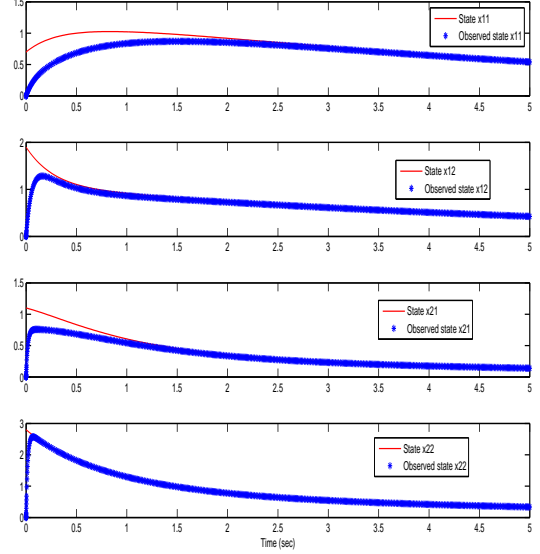

Fig. 1. States and observed states variation of subsystems 1 and 2

To determine the gain parameters of the decentralized observer, we choose a reference matrix described by:

$$M_r = \begin{pmatrix} -66 & 0 & 0 & 0 \\ 0 & -65 & 0 & 0 \\ 0 & 0 & -60 & 0 \\ 0 & 0 & 0 & -75 \end{pmatrix}$$

We have minimized the system of équations (28) using an orthogonal basis of $N = 10$ Legendre polynomials. The obtained gains are then expressed by the following vectors:

$$L_1 = \begin{pmatrix} 2.43 \\ 14.8 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 28 \\ 101 \end{pmatrix}$$

Figure 1 shows that the obtained state observer is able to track changes in subsystems states for any initial conditions, which proves its validity for the control of the considered interconnected system.

We aim now to determine a decentralized control law for each one of these subsystems such that they have an identical input-output dynamic evolution as the second order reference model characterized by the following matrices:

$$E = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} F = \begin{pmatrix} 2 \\ 0 \end{pmatrix} G = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

For this objective we have applied the proposed method using an orthogonal basis of $N = 10$ Legendre polynomials. The obtained decentralized laws are then characterized by the following gains:

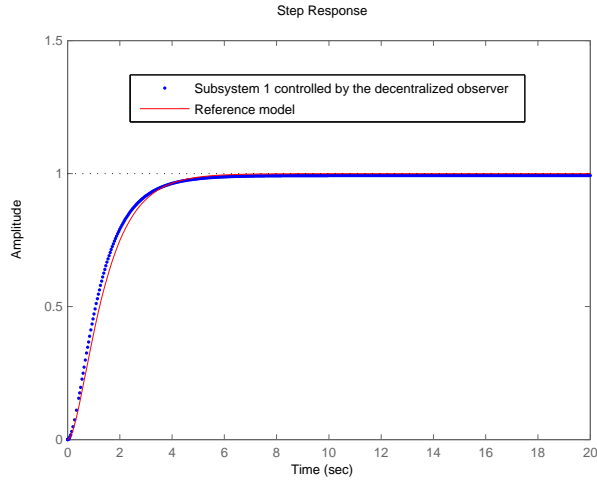$$K_1 = \begin{pmatrix} -0.4 & 2.8 \end{pmatrix} \quad J_1 = 2.8$$

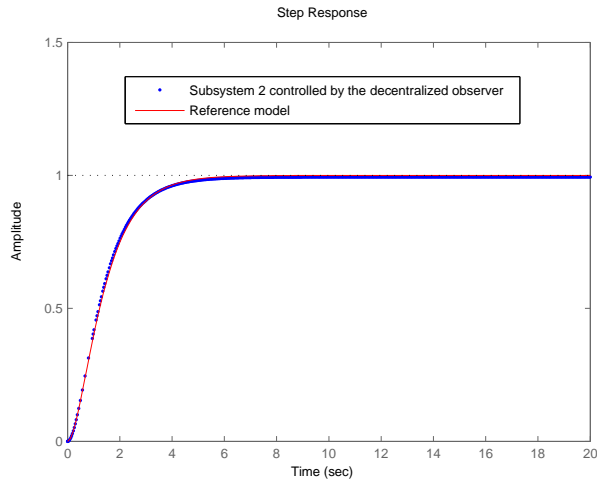Fig. 2. Step response of subsystem 1 and the reference model

Fig. 3. Step response of subsystem 2 and the reference model

$$K_2 = \begin{pmatrix} -1.1 & 0.8 \end{pmatrix} \quad J_2 = 3.2$$

Figure 2 and Figure 3 show the step reponses of the two interconnected systems with the obtained decentralized control laws, and the step response of the considered reference. Then, it appears clearly that the controlled system outputs are very close to the desired reference model output, which illustrates the validity of the proposed technique.

V. CONCLUSION

In this paper, a decentralized observer design has been developed. The proposed observer has been exploited to establish a decentralized control technique by using orthogonal functions as an interesting tool of dynamical system approximation. The main advantage of the proposed technique is its applicability for a large class of interconnected systems without imposing particular conditions. Furthermore, the decentralized controllers parameters are adjusted such that each subsystem has the specific desired performances of a chosen reference model. The validity of this new approach has been illustrated in a numerical example.

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